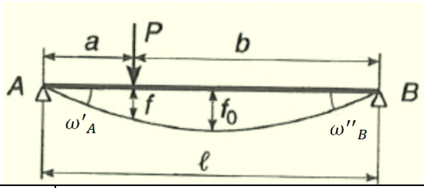
A. Poutres bi-appuyée

$R_A = \frac{Pb}{L}$

$\omega'_A = -\frac{Pa}{6EIL}(L-a)(2L-a)$



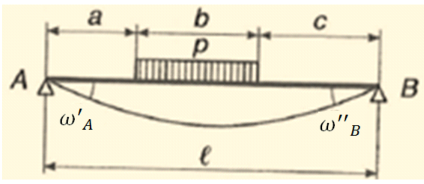
$R_B = \frac{Pa}{L}$

$\omega''_B = \frac{Pa}{6EIL}(L^2-a^2)$

Pour $0 < x < a$	$M(x) = \frac{Pbx}{L}$ $V(x) = \frac{Pb}{L}$ $f(x) = -\frac{Pbx}{6EIL}(L^2-b^2-x^2)$
Pour $a < x < L$	$M(x) = Pa\left(1-\frac{x}{L}\right)$ $V(x) = -\frac{Pa}{L}$ $f(x) = -\frac{Pa(L-x)}{6EIL}[x(2L-x)-a^2]$

$R_A = \frac{pb(b+2c)}{2L}$

$\omega'_A = -\frac{Pb(b+2c)}{48EIL}[4L^2-(b+2c)^2-b^2]$



$R_B = \frac{pb(2a+b)}{2L}$

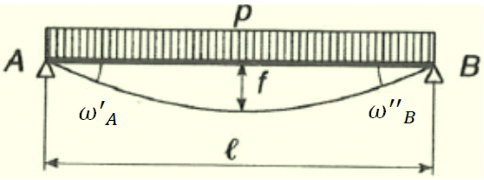
$\omega''_B = \frac{Pb(2a+b)}{48EIL}[4L^2-(2a+b)^2-b^2]$

Pour $0 < x < a$	$M(x) = \frac{pbx}{2L}(b+2c)$ $V(x) = \frac{pb}{2L}(b+2c)$
Pour $a < x < a+b$	$M(x) = \frac{pbx}{2L}(b+2c) - p\frac{(x-a)^2}{2}$ $V(x) = \frac{pb}{2L}(b+2c) - p(x-a)$ $f(x) = -\frac{p}{48EIL}[b(b+2c)x[4(l^2-x^2)-(b+2c)^2-b^2]+2l(x-a)^4]$
Pour $a+b < x < L$	$M(x) = \frac{pb}{2L}(2a+b)(L-x)$ $V(x) = -\frac{pb}{2L}(2a+b)$

$M_{max} = \frac{pb}{8L^2}(b+2c)[b(b+2c)+4aL]$ pour $x = a + \frac{b(b+2c)}{2L}$

$$\boxed{R_A = \frac{pL}{2}}$$

$$\boxed{\omega'_A = -\frac{pL^3}{24EI}}$$



$$\boxed{R_B = \frac{pL}{2}}$$

$$\boxed{\omega''_B = \frac{pL^3}{24EI}}$$

$$\boxed{M(x) = \frac{px}{2}(L - x)}$$

Avec $\boxed{M_{max} = \frac{pL^2}{8}}$ pour $x = \frac{L}{2}$

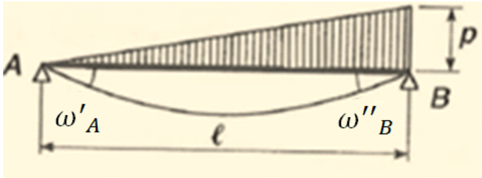
$$\boxed{V(x) = p\left(\frac{L}{2} - x\right)}$$

$$\boxed{f(x) = -\frac{px}{24EI}[L^3 - 2Lx^2 + x^3]}$$

Avec $\boxed{f_{max} = -\frac{5pL^4}{384EI}}$ pour $x = \frac{L}{2}$

$$\boxed{R_A = \frac{P}{3}}$$

$$\boxed{\omega'_A = -\frac{7pL^3}{360EI}}$$



$$\boxed{R_B = \frac{2P}{3}}$$

$$\boxed{\omega''_B = \frac{8pL^3}{360EI}}$$

$$\boxed{M(x) = \frac{Px}{3l^2}(L^2 - x^2)}$$

Avec $\boxed{M_{max} = \frac{2PL}{9\sqrt{3}}}$ pour $x = \frac{L}{\sqrt{3}}$

$$\boxed{V(x) = \frac{P}{3l^2}(L^2 - 3x^2)}$$

$$\boxed{f(x) = -\frac{Px}{180EIL^2}(L^2 - x^2)(7L^2 - 3x^2)}$$

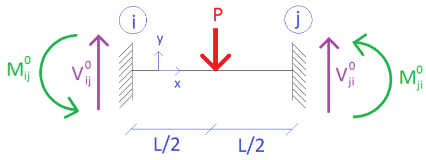
Avec $\boxed{f_{max} \approx -\frac{PL^3}{76{,}6EI}}$ pour $x \approx 0{,}519L$

B. Poutres bi-encastrées

Hypothèses :

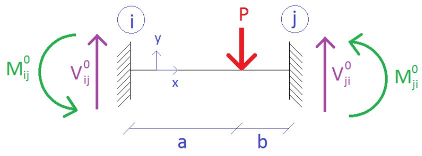
- EI est constante long de la poutre
- La longueur de la poutre est L

$$V_{ij}^0 = \frac{P}{2} \qquad M_{ij}^0 = \frac{PL}{8}$$



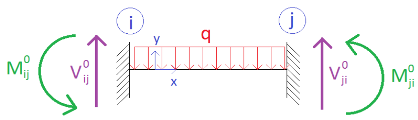
$$V_{ji}^0 = \frac{P}{2} \qquad M_{ji}^0 = -\frac{PL}{8}$$

$$V_{ij}^0 = \frac{Pb^2}{L^3}(3a+b) \qquad M_{ij}^0 = \frac{Pab^2}{L^2}$$



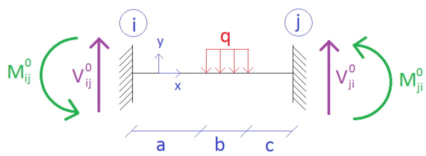
$$V_{ji}^0 = \frac{Pa^2}{L^3}(3b+a) \qquad M_{ji}^0 = -\frac{Pa^2b}{L^2}$$

$$V_{ij}^0 = \frac{qL}{2} \qquad M_{ij}^0 = \frac{qL^2}{12}$$



$$V_{ji}^0 = \frac{qL}{2} \qquad M_{ji}^0 = -\frac{qL^2}{12}$$

$$V_{ij}^0 = \frac{qb\left(c+\frac{b}{2}\right)^2}{L^3}\{3a+2b+c\}$$

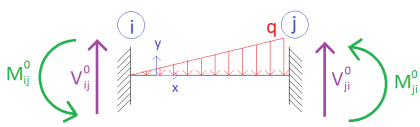


$$V_{ji}^0 = \frac{qb\left(a+\frac{b}{2}\right)^2}{L^3}\{3c+2b+a\}$$

$$M_{ij}^0 = \frac{qb}{24L^2}\cdot\left[(b+4c-2a)(4L^2-b^2)-2\cdot(b+2c)^3+(b+2a)^3\right]$$

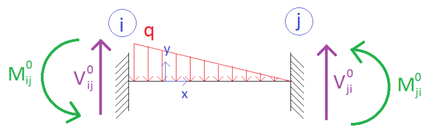
$$M_{ji}^0 = -\frac{qb}{24L^2}\cdot\left[(b+4a-2c)(4L^2-b^2)-2\cdot(b+2a)^3+(b+2c)^3\right]$$

$$V_{ij}^0 = \frac{3qL}{20} \qquad M_{ij}^0 = \frac{qL^2}{30}$$



$$V_{ji}^0 = \frac{7qL}{20} \qquad M_{ji}^0 = -\frac{qL^2}{20}$$

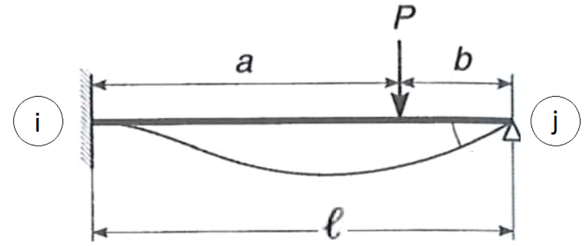
$$V_{ij}^0 = \frac{7qL}{20} \qquad M_{ij}^0 = -\frac{qL^2}{20}$$



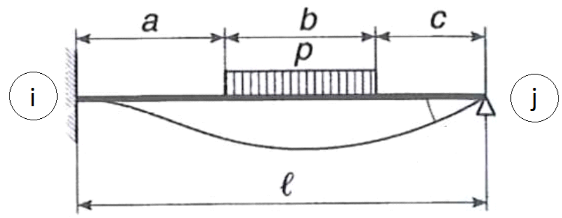
$$V_{ji}^0 = \frac{3qL}{20} \qquad M_{ji}^0 = \frac{qL^2}{30}$$

C. Moment d'encastrement des poutres sur un appui simple à droite et encastré à gauche

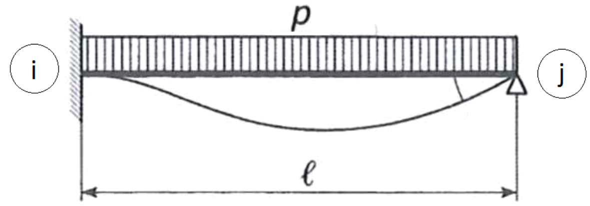
$$\boxed{M'_{ij} = -\frac{Pa}{2}\left(1-\frac{a}{l}\right)\left(2-\frac{a}{l}\right)}$$



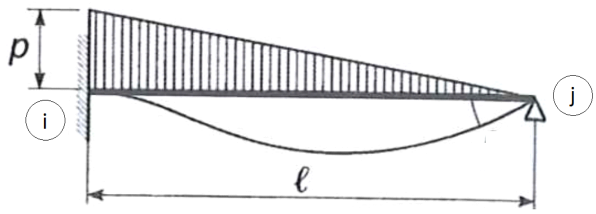
$$\boxed{M'_{ij} = -\frac{pl^2}{8}\left[1-\frac{c^2}{l^2}\left(2-\frac{c^2}{l^2}\right)-\frac{a^2}{l^2}\left(2-\frac{a}{l}\right)^2\right]}$$



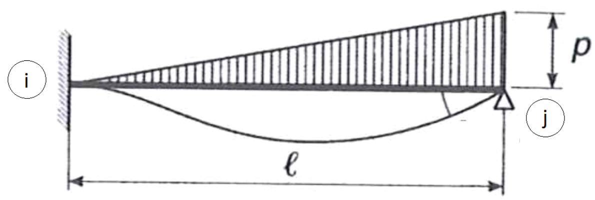
$$\boxed{M'_{ij} = -\frac{pl^2}{8}}$$



$$\boxed{M'_{ij} = -\frac{pl^2}{15}}$$

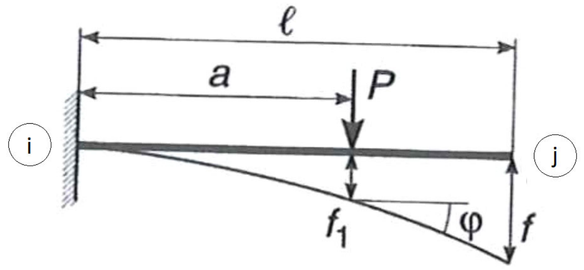


$$\boxed{M'_{ij} = \frac{7pl^2}{120}}$$

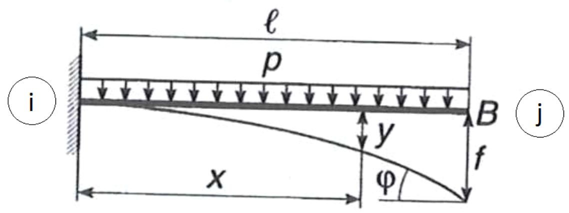


D. Moment d'encastrement dans une poutre console

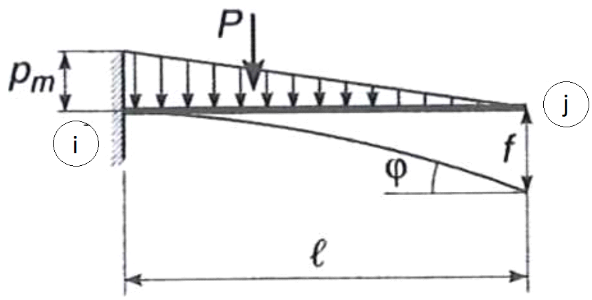
$$M''_{ij} = -Pa$$



$$M''_{ij} = -\frac{pl^2}{2}$$



$$M''_{ij} = -\frac{p_m l^2}{6}$$



$$M''_{ij} = -\frac{p_m l^2}{3}$$